AP CALCULUS AB — Homework 4.2a — Name:
DR. PAUL L. BAILEY — Tuesday, December 17, 2019

Write your homework *neatly, in pencil*, on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem*, or at least enough of it so that your work is readable. When appropriate, *write in sentences.*

The Mean Value Theorem is stated below.

Theorem 1. (Rolle's Theorem)
Let f be continuous on a closed interval $[a,b]$ and differentiable on (a,b). Suppose that $f(a) = f(b) = 0$. Then there exists $c \in (a,b)$ such that $f'(c) = 0$.

Theorem 2. (Mean Value Theorem (MVT))
Let f be continuous on a closed interval $[a,b]$ and differentiable on (a,b). Then there exists $c \in (a,b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Problem 1 (Thomas §4.2 # 4). Let $f(x) = \sqrt{x-1}$. Let $a = 1$ and $b = 3$. Find $c \in [a,b]$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Problem 2 (Thomas §4.2 # 5 - 8). Which functions satisfy the Mean Value Theorem on the indicated interval, and which do not? Justify you answer.

(a) $f(x) = x^{2/3}$ on $[-1, 8]$

(b) $f(x) = x^{4/5}$ on $[0, 1]$

(c) $f(x) = \sqrt{x(1-x)}$ on $[0, 1]$

(d) $f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{for } x \in [-\pi, 0) \\ 0 & \text{for } x = 0 \end{cases}$

Problem 3 (Thomas §4.2 # 10). Let

$$f(x) = \begin{cases} 3 & \text{for } x = 0 \\ -x^2 + 3x + a & \text{for } x \in (0,1) \\ mx + b & \text{for } x \in [1,2] \end{cases}$$

For what values of a, m, and b does f satisfy the hypothesis of the Mean Value Theorem on the interval $[0, 2]$?

Problem 4 (Thomas §4.2 # 15). Show that the function

$$f(x) = x^4 + 3x + 1$$

has exactly one zero on $[-2, -1]$.

Problem 5 (Thomas §4.2 # 19). Show that the function

$$r(\theta) = \theta + \sin^2(\theta/3) - 8$$

has exactly one zero on $\mathbb{R}$.

Problem 6. Find all real zeros of the following polynomials.

(a) $f(x) = x^3 + 3x^2 - 4x - 12$ (Factor by Grouping)

(b) $g(x) = x^4 - 2x^2 - 15$ (Factor by Substitution $u = x^2$)

Problem 7. Find all real zeros of the following polynomials.

(a) $p(x) = x^3 - 4x^2 - 11x + 30$ (Integer Zeros Theorem)

(b) $q(x) = 3x^3 + 11x^2 - 19x + 5$ (Rational Zeros Theorem)

Problem 8 (Thomas §3.6 # 46)**.** Consider the equation

$$(x^2 + y^2)^2 = (x - y)^2.$$

Problem 9 (Thomas §3.8 # 27)**.** A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x-coordinate (measured in meters) increases at a steady 10 m/sec. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = 3$ m?

Problem 10 (Thomas §4.1 #4)**.** Let

$$f(x) = \frac{x+1}{x^2 + 2x + 2}.$$

Find all local extreme values of the function f, and where they occur.